

ADDING & SUBTRACTING FLUENTLY

described by Courtney Flessner and Ryan Flessner

Often, fluency in arithmetic is understood to mean that students can use the standard U.S. algorithms to achieve correct answers quickly. Unfortunately, simply mastering the steps of the standard U.S. algorithms does not ensure that students have a deep understanding of the concepts of multi-digit addition and subtraction or adding and subtracting with fractions or decimals. The standards in this strand allow students to build conceptual understandings in a variety of ways prior to learning and mastering the standard U.S. algorithms. While the expectation for learning these algorithms remains, the standards in this progression allow students to understand why the algorithms work by first constructing concrete understandings, moving to strategies that connect to understandings of place value and the properties of operations, and then to mastering the standard U.S. algorithms.

KINDERGARTEN

Use objects, drawings, mental images, sounds, etc., to represent addition and subtraction within 10. (K.CA.1)

Unpacking the Standard

Young children are constantly sorting, combining, and separating items. This work leads to the concepts of addition and subtraction. In Kindergarten, students make sense of combinations within 10. They use pictures, manipulatives, their senses, their bodies, and other tools to represent these combinations.

Considerations for Lessons and Assessment

Those who work with kindergarteners should provide students with multiple entry points when considering addition and subtraction within 10. These include manipulatives (e.g., counting bears, unifix cubes, two color counters), drawings, mathematical structures (e.g., ten frames, number lines), sounds, and physical movement. Assessments should be informal in nature as teachers observe children interacting informally, working with concrete materials, and drawing pictures of their mathematical understandings.

GRADE 1

VIDEO 1 OF 2

Demonstrate fluency with addition facts and the corresponding subtraction facts within 20. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a 10 (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$). Understand the role of 0 in addition and subtraction. (1.CA.1)

Unpacking the Standard

First graders should master basic addition and subtraction facts to 20 by the end of the year. As students come to understand these ideas, educators should ensure that students are fluent with their facts. This entails both correct answers and an explanation of how the answer was achieved (e.g., students use a known fact such as $7+3=10$ to solve a related fact such as $7+4$; students use a known addition fact such as $3+8=11$ to solve the subtraction fact $11-8=3$).

Considerations for Lessons and Assessment

Lessons in first grade should encourage students to make connections between known facts and those facts they are coming to understand. Memorization and timed assessments should be avoided when assessing standard 1.CA.1. Instead, educators can use a variety of assessments (e.g., observations, interviews, or student-created drawings) to ensure that students are flexible, efficient, and accurate.

VIDEO 2 OF 2

Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; describe the strategy and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones, and that sometimes it is necessary to compose a ten. (1.CA.5)

Unpacking the Standard

Building on their knowledge of basic facts, students in first grade extend this knowledge to addition problems with sums as high as 100. Within this range, students should be able to add a two-digit number and a one-digit number ($58+3$) as well as a two-digit number and a multiple of 10 ($58+30$). Problems such as $58+33$ should be reserved for second grade.

Considerations for Lessons and Assessment

Lessons related to this standard should focus on multiple representations. Educators should avoid abstract methods such as the standard U.S. algorithms and, instead, focus on tools and strategies such as unifix cubes (in groups of 5s and 10s), hundreds charts, and base ten blocks. Assessments should engage children in achieving correct answers while verifying that students can explain their chosen strategies.

GRADE 2

Add and subtract fluently within 100. (2.CA.1)

Unpacking the Standard

Fluency in addition and subtraction requires students to demonstrate flexibility, efficiency, and accuracy when solving problems. Second graders are expected to build on knowledge they learned in first grade (i.e., counting on, making tens, knowledge of addition and subtraction facts within 20) as problems increase in complexity.

Considerations for Lessons and Assessment

This video explores ways that students can utilize tools such as base ten blocks and number lines to conceptualize addition and subtraction within 100. In order to assess understanding, formal and informal assessments should ask students to utilize strategies to add and subtract within 100; however, assessments must have the dual purpose of confirming that students can explain the strategies they use. This video examines ways teachers can ensure students exhibit both correct answers and correct thinking.

GRADE 3

Fluently add and subtract whole numbers within 1000 using strategies and algorithms based on place value, properties of operations, and relationships between addition and subtraction. (3.C.1)

Unpacking the Standard

Building on concepts explored in second grade, this video highlights the move to larger numbers (up to and within 1,000) and more abstract strategies and notation systems in third grade. Properties of operations (commutative, associative, distributive) are introduced, and students are encouraged to use what they know about addition problems to solve subtraction problems (and vice versa). It should be noted that the standard U.S. algorithms for addition and subtraction are not included in the third-grade standard. While they may be utilized by third graders who are ready to engage with these algorithms conceptually, they should not be expected of all third graders or taught as whole class lessons as this is the fourth-grade expectation.

Considerations for Lessons and Assessment

Strategies and algorithms explored in this video include more efficient uses of base ten blocks and number lines as well as an introduction to the pull down method, the partial sums/differences algorithms, arrow language, and number sentence strings. Informal and formal assessments should ensure that students' fluency (flexibility, efficiency, and accuracy) move developmentally from the concrete to the more abstract as students progress throughout their third-grade year.

GRADE 4

VIDEO 1 OF 2

Add and subtract multi-digit whole numbers fluently using a standard algorithmic approach. (4.C.1)

Unpacking the Standard

Fourth grade is when students should master the standard U.S. algorithms in addition and subtraction. However, it is expected that students' prior understandings from previous grades are used as the starting point in fourth grade.

Considerations for Lessons and Assessment

Strengths exhibited through the use of base ten blocks, number lines, and strategies based in place value or the properties of operations (explored in prior grades) are the foundation upon which fourth-grade teachers should build. This video shows how connecting these ideas to the standard U.S. algorithm will ensure that fourth graders build conceptual understandings that allow them to achieve correct answers while thinking deeply about the mathematics involved. Assessments in fourth grade should encourage students to communicate the ways they are connecting what they know to new ideas they are exploring.

VIDEO 2 OF 2

Add and subtract fractions with common denominators. Decompose a fraction into a sum of fractions with common denominators. Understand addition and subtraction of fractions as combining and separating parts referring to the same whole. (4.C.5)

Unpacking the Standard

Addition and subtraction of fractions is a new concept in fourth grade. As such, educators should ensure that students have concrete experiences with the problems. Furthermore, problems should be limited to adding and subtracting fractions with common denominators. More complex problems (i.e., uncommon denominators, mixed numbers) are introduced in fifth grade. Utilizing the standard U.S. algorithms when adding or subtracting fractions should be reserved for students in sixth grade.

Considerations for Lessons and Assessment

Manipulatives such as fraction bars and pattern blocks are excellent concrete materials for adding and subtracting fractions. These materials allow educators to capitalize on the “parts referring to the same whole” aspect of this standard. Fractional parts such as halves, thirds, fourths, sixths, and tenths could possibly connect to students’ prior experiences (e.g., baking, measuring, counting change, telling time). Because of this, items such as rulers, measuring tapes, and other number line-like tools should be accessible to students. Assessments should allow students to utilize concrete materials in order to communicate their knowledge.

GRADE 5

VIDEO 1 OF 2

Add, subtract, multiply, and divide decimals to hundredths, using models or drawings and strategies based on place value or the properties of operations. Describe the strategy and explain the reasoning. (5.C.8)

Unpacking the Standard

In fifth grade students will expand their understandings of addition and subtraction by engaging with numbers containing decimals and reviewing the properties of operations (commutative, associative, distributive) introduced in third grade.

It is important to note that this strand focuses on addition and subtraction. For ideas related to multiplication and division with decimals to hundredths, please see the “Multiplying and Dividing Fluently” series.

Considerations for Lessons and Assessment

This video explains that by stressing place value understandings and properties of operations, this standard encourages educators to return to less abstract tools and notation systems (base ten blocks, number lines, pull down method, partial sums/differences, arrow language, number sentence strings) instead of simply applying the standard U.S. algorithms to numbers with decimals (which should be mastered by the end of sixth grade). As in prior grades, formal and informal assessments in fifth grade should ensure that students demonstrate both correct answers and correct thinking.

VIDEO 2 OF 2

Add and subtract fractions with unlike denominators, including mixed numbers. (5.C.4)

Unpacking the Standard

In fifth grade students continue working with addition and subtraction of fractions by building on their knowledge from the previous year. New twists in fifth grade include the introduction of fractions with unlike denominators as well as addition and subtraction with mixed numbers. Educators should continue to refrain from whole class lessons that teach standard U.S. algorithms to fifth graders as this concept is reserved for sixth grade. Instead, fifth-grade students should build conceptual understandings of these concepts that they can communicate effectively.

Considerations for Lessons and Assessment

Building on students' experiences with concrete materials and with common denominators in fourth grade, educators working with fifth graders should engage students in discovering similar connections with fractions containing unlike denominators. A variety of materials (e.g., fraction bars and circles, pattern blocks, money, number lines) can assist students in making sense of these relationships. Assessments should ensure that students use words, pictures, and numbers to demonstrate their knowledge of the concepts under review.

GRADES 6-7

Compute with positive fractions and positive decimals fluently using a standard algorithmic approach. (6.C.2)

Compute fluently with rational numbers using an algorithmic approach. (7.C.7)

Unpacking the Standards

In Grades 6 and 7 students are formally introduced to the standard U.S. algorithms when working with fractions and decimals. This work should build on the conceptual understandings students bring from prior grades.

It is important to note that this strand focuses on addition and subtraction. For ideas related to multiplication and division with decimals to hundredths, please see the "Multiplying and Dividing Fluently" series.

Considerations for Lessons and Assessment

Rather than starting with standard U.S. algorithms, educators in Grades 6 and 7 should connect these ideas to the concrete representations students bring from prior mathematical experiences. Assessments should ask students to solve problems in multiple ways in order to strengthen students' connections between concrete and abstract strategies. Furthermore, educators should look for both correct answers and correct thinking as students solve problems.

ADDITIONAL RESOURCES

- Bay-Williams, J. M., & SanGiovanni, J. J. (2021). *Figuring out fluency in mathematics teaching and learning, grades K-8: Moving beyond basic facts and memorization*. Thousand Oaks, CA: Corwin.
- Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2014). *Children's mathematics: Cognitively guided instruction* (2nd ed.). Portsmouth, NH: Heinemann.
- Chapin, S. H., & Johnson, A. (2006). *Math matters: Understanding the math you teach, grades K-8* (2nd ed.). Sausalito, CA: Math Solutions Publications.
- O'Connell, S., & SanGiovanni, J. (2011). *Mastering the basic math facts in addition and subtraction: Strategies, activities, and interventions to move students beyond memorization*. Portsmouth, NH: Heinemann.
- Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2018). *Elementary and middle school mathematics: Teaching developmentally* (10th ed.). Boston, MA: Pearson.
- Partnership for Inquiry Learning (n.d.). <http://partnershipforinquirylearning.org>.

This document and accompanying videos may be shared in their original form for educational purposes only. No modifications or reproductions permitted without consent of Partnership for Inquiry Learning and the authors.