MULTIPLYING AND DIVIDING FLUENTLY

described by Courtney Flessner and Ryan Flessner

There are often misconceptions that fluency means one must memorize basic facts and standard U.S. algorithms in order to do math quickly. Such is not the case. Students are fluent when they can efficiently solve a problem using strategies they understand. Further, students are fluent when they build off their number sense and explain how they computed. As students achieve correct answers, they must do so while explaining their methods. The standards in this strand allow students to transition from skip counting to learning and mastering basic multiplication and division facts. After that, they expand their fact knowledge into fluently multiplying and dividing within 100 before they master the standard U.S. algorithm for multiplication in Grade 5 and for division in Grade 6. The expectation is that students will understand why the standard U.S. algorithms for multiplication and division work by building off prior knowledge from Grades 2-4. Additionally, students begin multiplying and dividing fractions in Grade 5. Initially, they engage in concrete experiences that foster conceptual understandings as they move toward the standard U.S. algorithms in Grade 6. In seventh grade students expand upon prior knowledge from Grades K-6 to fluently multiply and divide rational numbers.

KINDERGARTEN - GRADE 1

Count to at least 100 by ones and tens and count on by one from any number. (K.NS.1)

Count to at least 120 by ones, fives, and tens from any given number. In this range, read and write numerals and represent a number of objects with a written numeral. (1.NS.1)

Unpacking the Standards

In kindergarten and first grade, students should regularly engage in skip counting. While the standards state that students should count by ones and tens (K) and ones, fives, and tens (1st), the key is to start from *any given number* to at least 100 or 120. For example, we want them to be able to start at 38 and count: 38, 48, 58, 68, etc. In first grade students should continually practice this skill as they move across 100.

Considerations for Lessons and Assessment

This video shows how students can skip count in a large or small group, by any given number, on a regular basis. Choral counting is presented as a way for students to practice their skip counting. Formative and summative assessments include individual and small group interviews, observations during whole group skip counting, observing number patterns, and filling in hundreds charts.

GRADE 2

Count by ones, twos, fives, tens, and hundreds up to at least 1,000 from any given number. (2.NS.1)

Unpacking the Standard

In second grade students should regularly practice counting and skip counting. Counting can start from the number itself (e.g. 5, 10, 15, 20…), but students should also regularly count from any number (e.g. 3, 8, 13, 18, 23…). In doing this, they will begin to see numerical patterns and develop an understanding of how the patterns change when a second or third digit is introduced to the number.

Considerations for Lessons and Assessment

Similar to K-1, this video shows how students can skip count in a large or small group, by any given number, on a regular basis. Choral counting will be introduced as well as how students can practice their skip counting while counting collections. Formative and summative assessments will include individual and small group interviews, observations during whole group skip counting, observing number patterns and filling in hundreds charts.

GRADE 3

Demonstrate fluency with mastery of multiplication facts and corresponding division facts of 0 to 10. (3.C.6)

Unpacking the Standard

Fluency in multiplication and division requires students to demonstrate flexibility, efficiency, and accuracy when solving problems. Students need to spend their third-grade year working on multiplication facts, developing an understanding of the relationship between multiplication and division, and then using that knowledge to master basic division facts.

Considerations for Lessons and Assessment

This video introduces activities you can do to build fluency with your students. It also shows how asking questions of your students while observing their play will help you understand what facts they know, the strategies they are using, and how they might be able to use those strategies to work on additional facts. Additionally, it shows how you can develop a clear understanding of the relationship between multiplication and division.

GRADE 4

VIDEO 1 OF 2

Multiply fluently within 100. (4.C.4)

Unpacking the Standard

Prior to fourth grade, students work to develop their fluency and master their basic facts up to 10x10 and 100/10. They have also determined what strategies work best for them to efficiently solve a problem. Using this knowledge as well as their place value number sense, students move into fluently multiplying and dividing two-digit numbers by one-digit numbers (e.g. 3x32 and 75/5).

Considerations for Lessons and Assessment

This video shows how students can build on their knowledge of basic facts, as well as number sense they have developed throughout their elementary career, to multiply increasingly larger numbers. Additionally, they will be able to explain their thinking to show a true conceptual understanding of their processes. Lessons and assessments should teach and evaluate students' developing understanding of strategies and tools used for multiplying and dividing (e.g., base ten blocks, ratio tables, partial products, and area models).

VIDEO 2 OF 2

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Describe the strategy and explain the reasoning. (4.C.3)

Unpacking the Standard

Building on their number sense from the early grades as well as the basic math facts they mastered in third grade, fourth graders begin to explore multi-digit division. It is imperative that educators working with fourth graders resist the temptation to teach the standard U.S. algorithm as this is a *sixth-grade* standard (6.C.1). Prior to an introduction to the standard U.S. algorithm, students must build conceptual understandings through concrete experiences.

Considerations for Lessons and Assessment

Lessons in Grade 4 should highlight a variety of tools students can use to divide multi-digit numbers. These could capitalize on knowledge of addition, subtraction, and multiplication and utilize tools such as base ten blocks and ratio tables. Assessments should ask students to solve problems with multiple strategies while also expecting students to understand and explain the mathematics behind the strategies they use.

GRADE 5

VIDEO 1 OF 5

Multiply multi-digit whole numbers fluently using a standard algorithmic approach. (5.C.1)

Unpacking the Standard

In fifth grade students master the standard U.S. algorithm. It is expected that students' prior understandings from previous grades are used as the starting point in fourth grade. Instruction related to the standard U.S. algorithm should build upon these prior understandings.

Considerations for Lessons and Assessment

This video will demonstrate how knowledge developed in fourth grade can be leveraged to efficiently master the standard U.S. algorithm. It will show an inquiry based approach to breaking down the standard U.S. algorithm so that students are learning, not just memorizing, procedural steps. Formative assessments will allow opportunities for students to demonstrate their understanding of the algorithm using smaller numbers. Summative assessments will allow students to solve multi-digit by multi-digit problems. Find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.

VIDEO 2 OF 5

Describe the strategy and explain the reasoning used. (5.C.2)

Unpacking the Standard

In fourth grade students divided four-digit dividends by one-digit divisors. The fifth-grade standard extends this work to include division with two-digit divisors. Educators should continue to refrain from whole-class lessons related to the standard U.S. algorithm as this is a *sixth-grade* standard (6.C.1).

Considerations for Lessons and Assessment

Lessons in Grade 5 should build on students' concrete experiences and conceptual understandings from fourth grade. This means moving students beyond work with manipulatives such as base ten blocks to more efficient strategies such as ratio tables, area models, and the partial quotients algorithm. Assessments should capitalize on the use of multiple strategies while stressing communication of conceptual understandings (as opposed to following procedures dictated by the teacher).

VIDEO 3 OF 5

Use visual fraction models and numbers to multiply a fraction by a fraction or a whole number. (5.C.5)

Unpacking the Standard

Multiplication of fractions is a new concept in fifth grade. The standard focuses educators' attention on concrete ideas that will lead to conceptual understandings. The idea is to promote understanding over rote memorization.

Considerations for Lessons and Assessment

Visual fraction models include fractions bars and circles, pattern blocks, number lines, and other mathematical tools. Visual fraction models allow students to explore multiplication of fractions in concrete ways while connecting these ideas to prior knowledge. Informal assessments such as observations and interviews can assist educators in pushing students' thinking on a daily basis. Summative assessments should look for correct answers while also insisting that students communicate *how* they achieved these answers.

VIDEO 4 OF 5

Use visual fraction models and numbers to divide a unit fraction by a non-zero whole number and to divide a whole number by a unit fraction. (5.C.7)

Unpacking the Standard

When dividing fractions in fifth grade, teachers should focus students' attention on conceptual understandings over rote memorization of procedures. Students should strive to build strong understandings of concrete, visual, and pictorial representations before they are introduced to the standard U.S. algorithm in sixth grade. Educators should take special note of this standard's focus on division of, or by, a unit fraction.

Considerations for Lessons and Assessment

As with the multiplication of fractions standard, visual fraction models (e.g., fraction circles, fractions bars, pattern blocks, number lines) provide students opportunities to develop strong concrete, visual, and pictorial representations when dividing fractions. Lessons should build on students' number sense related to fractions. Teachers can use observations and one-on-one conversations to informally assess students' knowledge as they use manipulatives and other tools to grow their understandings. Formal assessments should remain focused on visual representations of division with fractions.

VIDEO 5 OF 5

Add, subtract, multiply, and divide decimals to hundredths, using models or drawings and strategies based on place value or the properties of operations. Describe the strategy and explain the reasoning. (5.C.8)

Unpacking the Standard

Students in fifth grade begin to explore multiplication and division with decimals. It is important to note that the newness of these ideas demands that students focus on conceptual understandings rather than simply applying rote procedures to these problems. Even though fifth graders are introduced to the standard U.S. algorithm for multiplication (5.C.1), it is important for educators to remember that this algorithm should not yet be translated to multiplication with decimals. Similarly, the standard U.S. algorithm for division is not formally introduced until Grade 6 (6.C.1); therefore, educators should avoid its use in relation to this standard.

It is important to note that this strand focuses on multiplication and division. For ideas related to addition and subtraction with decimals to hundredths, please see the "Adding and Subtracting Fluently" series.

Considerations for Lessons and Assessment

Number sense skills (e.g., estimation and place value understandings), concrete materials (e.g., base ten blocks), and multiple strategies (e.g., ratio tables, the area model, and partial products/quotients) allow for deep understandings related to multiplication and division with decimals. Assessments should encourage correct answers *and* correct thinking. Students should be given a wide array of opportunities (written and oral) to explain their conceptual understandings related to multiplication and division with decimals.

GRADE 6

Divide multi-digit whole numbers fluently using a standard algorithmic approach. (6.C.1)

Unpacking the Standard

Fourth- and fifth-grade students spend significant amounts of time understanding the concept of division conceptually. In sixth grade this prior knowledge is crucial. Rather than negating all of this prior knowledge, educators in Grade 6 should capitalize on these understandings when introducing the standard U.S. algorithm.

Considerations for Lessons and Assessment

Rather than teaching rote procedures for students to memorize, educators in Grade 6 should ensure that students understand *how* and *why* the standard U.S. algorithm works. This can be done by relating the algorithm to conceptual understandings from previous grades. Concrete models like base ten blocks and strategies such as the partial quotients algorithm provide solid foundations upon which educators in Grade 6 can build. When assessing this standard, great care should be taken to ensure that students are not simply regurgitating procedures delineated by adults. Students should be able to explain the algorithm while connecting it to mathematical knowledge that justifies each move they make.

GRADES 6-7

Compute with positive fractions and positive decimals fluently using a standard algorithmic approach. (6.C.2)

Compute fluently with rational numbers using an algorithmic approach. (7.C.7)

Unpacking the Standards

Building on concrete experiences, conceptual understandings, and strategies students learned in Grade 5, sixth and seventh graders extend their knowledge of multiplication and division as they work with the standard U.S. algorithms related to fractions, decimals, and other rational numbers.

It is important to note that this strand focuses on multiplication and division. For ideas related to addition and subtraction with decimals to hundredths, please see the "Adding and Subtracting Fluently" series.

Considerations for Lessons and Assessment

Lessons for sixth and seventh graders should capitalize on prior understandings as standard U.S. algorithms are introduced. A premium must be placed on students' understandings. Teaching rote procedures negates the habits of mind students have demonstrated in earlier grades. Connections between visual models used in the teaching of fractions and decimals should be made as students explore more abstract ideas in Grades 6 and 7. Assessments requiring the use of standard algorithms should also encourage students to employ other strategies in order to demonstrate depth of understanding. Educators should look for both correct answers and correct thinking in order to affirm that students can justify the mathematics involved with any strategies they use.

ADDITIONAL RESOURCES

Bay-Williams, J. & Kling, G. (2019). *Math fact fluency: 60+ games and assessment tools to support learning and retention.* ASCD.

Bay-Williams, J. M., & SanGiovanni, J. J. (2021). *Figuring out fluency in mathematics teaching and learning: Moving beyond basic facts and memorization*. Corwin.

Burns, M. (2015). *About teaching mathematics: A K-8 resource (4th ed.)*. Math Solutions.

Franke, M.L., Kazemi, E., & Turro, A.C. (2018). *Choral counting and counting collections.* Stenhouse.

Kling, G. & Bay-Williams, J. (2018). *Games and tools for teaching multiplication facts (a quick reference guide).* ASCD.

O'Connell, S. & SanGiovanni, J. (2014). *Mastering the basic math facts in multiplication and division: Strategies, activities, and interventions to move students beyond memorization*. Heinemann.

Partnership for Inquiry Learning (n.d.). http://partnershipforinquirylearning.org.